

JEE QUESTION: SOLIDS

- 1) A wire of length l & cross-sectional area A is suspended. A mass M is hung from it causing an extension x . If the wire is replaced by another wire of same material but double the length & half the diameter, find the extension under the same load

ANSWER

$$x = \frac{FL}{AY} \quad x' = \frac{FL'}{A'Y} = \frac{F(2L)}{(A/4)Y} = 8 \frac{FL}{AY}$$

$$A = \pi R^2$$

$$L' = 2L$$

$$R' = \frac{R}{2}$$

$$A' = \pi \left(\frac{R}{2}\right)^2 = \frac{A}{4}$$

$$\Rightarrow x' = 8x$$

- 2) A metal rod of length l & radius R is stretched by a tensile force F . Its Poisson's ratio is σ . Find the fractional change in volume

ANSWER

Longitudinal strain:

$$\epsilon_l = \frac{\Delta l}{l} = \frac{F}{YA}$$

Lateral strain:

$$\epsilon_t = -\sigma \epsilon_l$$

Fractional change in volume:

$$\frac{\Delta V}{V} \approx \epsilon_l + 2\epsilon_t = \epsilon_l - 2\sigma \epsilon_l = \epsilon_l (1 - 2\sigma)$$

$$\Rightarrow \frac{\Delta V}{V} = \frac{F}{YA} (1 - 2\sigma)$$

3) Two wires of different materials, lengths L_1, L_2 & cross sectional areas A_1, A_2 are joined in series. Young's moduli $= Y_1, Y_2$. A load F is applied. Find the total extension of the system.

ANSWER

$$\Delta L_1 = \frac{FL_1}{Y_1 A_1}, \Delta L_2 = \frac{FL_2}{Y_2 A_2}$$

$$\Delta L_{\text{total}} = \Delta L_1 + \Delta L_2 = \frac{FL_1}{A_1 Y_1} + \frac{FL_2}{A_2 Y_2}$$

$$\Rightarrow \Delta L_{\text{total}} = \frac{FL_1}{Y_1 A_1} + \frac{FL_2}{A_2 Y_2}$$

Q A sample of a liquid is kept at 1 atm. It is compressed to 5 atm which leads to a change of volume of 0.8 cm^3 . If the bulk modulus of the liquid is 2 GPa, the initial volume of the liquid was — litre (Take $1 \text{ atm} = 10^5 \text{ Pa}$)

Sol: Given, initial pressure of liquid (P_1) = 1 atm

Final pressure of liquid (P_2) = 5 atm

$$\text{Change in pressure } (\Delta P) = P_f - P_i = 4 \text{ atm} \\ = 4 \times 10^5 \text{ Pa}$$

$$\text{Change in volume } (\Delta V) = -0.8 \text{ cm}^3$$

$$\text{Bulk modulus} = 2 \times 10^9 \text{ Pa}$$

$$\text{Now, } B = \frac{-\Delta P}{(\Delta V/V)} \Rightarrow V = -B \left(\frac{\Delta V}{\Delta P} \right)$$

$$\Rightarrow V = -2 \times 10^9 \times \frac{(-0.8 \times 10^{-6})}{4 \times 10^5}$$

$$= 4 \times 10^{-3} \text{ m}^3 = 4 \text{ litre}$$

Q The length of a light string is 1.4 m when the tension on it is 5 N. If the tension increases to 7 N, the length of the string is 1.56 m. The original length of the string is — m

Sol: To find original length of the string, we use relationship between tension, elasticity constant (K) and change in length of the string

$$T = K(l - l_0)$$

$l \rightarrow$ length of string under tension, $l_0 \rightarrow$ original length

when $T = 5 \text{ N}$

$$5 = K(1.4 - l_0)$$

when $T = 7 \text{ N}$

$$7 = K(1.56 - l_0)$$

by setting up a ratio from the two equations

$$\frac{5}{1.4 - l_0} = \frac{7}{1.56 - l_0}$$

$$5(1.56 - l_0) = 7(1.4 - l_0)$$

$$7.8 - 5l_0 = 9.8 - 7l_0$$

$$2l_0 = 2$$

$$l_0 = 1\text{m}$$

$\therefore$ Original length of the string is 1m

Q Young's modulus of material of a wire of length 'L' and cross sectional area A is Y. If the length of the wire is doubled and cross sectional area is halved, then Young's modulus will be

Sol: Young's modulus depends on the material not length and cross sectional area. so young's modulus remains same.

Mechanical Properties OF Solids

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JEE Questions

①

Q.1) Two wires A and B are made of some material having ratio of lengths $\frac{L_A}{L_B} = \frac{1}{3}$ and their diameters ratio $\frac{d_A}{d_B} = 2$.

If both the wires are stretched using same force, what would be the ratio of their respective elongations?

[JEE Main 2025]

(A) 3:4

(B) 1:12

(C) 1:3

(D) 1:6

Ans: (B) $3L_A = L_B$; $d_A = 2d_B$

Given that: - Same material, meaning their Young modulus are equal $\{Y_A = Y_B\}$.

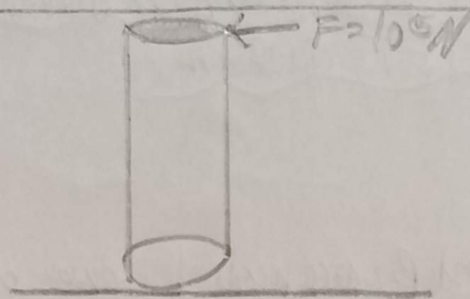
$$\Rightarrow Y_A = Y_B$$

$$\Rightarrow \frac{F/L_A}{A \times \Delta L_A} = \frac{F/L_B}{A \times \Delta L_B}$$

$$\frac{\Delta L_A}{\Delta L_B} = \left(\frac{L_A}{L_B}\right) \left(\frac{d_B}{d_A}\right)^2 \Rightarrow \left(\frac{1}{3}\right) \left(\frac{1}{2}\right)^2 \Rightarrow \frac{1}{3} \times \frac{1}{4}$$
$$\Rightarrow \frac{1}{12} \Rightarrow \underline{\underline{1:12}} \text{ (B)}$$

Q.2) A cylindrical rod of length 1m and radius 4cm is mounted vertically. It is subjected to a shear force of 10^5 N at the top. Considering infinitesimally small displacement in the upper edge, the angular displacement θ of the rod axis from its original position would be: - ($G = 10^{10}$ N/m²)

Ans: - (*)



(2)

Shear moduli $\Rightarrow \frac{\sigma_{\text{shear}}}{\theta}$

$$10^{10} = \frac{10^8}{\pi \times 16 \times 10^{-4}} \times \frac{1}{\theta}$$

$$\therefore \theta = \frac{1}{160\pi} \text{ radian}$$

Q.3) Young's modulus of material of a wire of length 'L' and cross-sectional area A is Y . If the length of wire is doubled and cross-sectional area is halved then 'Y' will be?

(A) $4Y$

(B) $2Y$

(C) $Y/4$

(D) Y

Ans: - (*) The answer is (D) Y , as young's modulus does not depend cross-sectional area and length. It depends on the material of the rod. So, it remains the same.

Mechanical Properties of Solids

1. Calculate structural compressive strain under a massive load. For a structural steel column (radius 0.3m inner, 0.6m outer) bearing a 50,000 kg load (Mg), the compressive strain is $\sim 7.22 \times 10^{-7}$.

$$\Rightarrow \text{Mass, } M = 50000 \text{ kg} ; g = 9.8 \text{ m/s}^2 ; \text{Young's Modulus, } Y = 2 \times 10^{11} \text{ Pa} ; \text{Inner radius } r = 0.3 \text{ m} ; \text{Outer radius, } R = 0.6 \text{ m}$$

$$\text{Total force} = Mg = 50000 \times 9.8 \\ = 490000 \text{ N}$$

$$\text{Force per column} = \frac{Mg}{4} = \frac{490000}{4} = \underline{\underline{122,500 \text{ N}}}$$

$$\text{Area, } A = \pi(R^2 - r^2) = \pi(0.6^2 - 0.3^2) \\ = \pi(0.36 - 0.09) = 0.27\pi \text{ m}^2$$

$$\text{Stress } (\sigma) = \frac{F}{A} = \frac{122500}{0.27\pi} = \frac{122500}{0.848} \approx \underline{\underline{144,457 \text{ Pa}}}$$

$$\text{Strain } (\epsilon) = \frac{\sigma}{Y} = \frac{144457}{2 \times 10^{11}} \approx \underline{\underline{7.22 \times 10^{-7}}}$$

$\therefore$ Final Result $\Rightarrow$ The compressive strain is $\underline{\underline{7.22 \times 10^{-7}}}$

2. A copper wire of length L and radius r is suspended vertically from a rigid support. If a mass m is attached to the free end of the wire, find the increase in length of the wire. Given: Young's modulus of copper is Y .

$$\Rightarrow F = mg$$

$$\text{Stress} = \frac{F}{A} = \frac{mg}{\pi r^2}$$

$$\text{Strain} = \frac{\Delta L}{L}$$

$$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{mg}{\pi r^2} \times \frac{L}{\Delta L}$$

$$\underline{\underline{\Delta L = \frac{mgL}{\pi r^2 Y}}}$$

3. A steel wire of length L , cross-sectional area A , and Young's modulus Y is stretched by a force F . If the wire is stretched by a distance x , derive an expression for the elastic potential energy stored in the wire.

$\Rightarrow$ Hooke's Law

$$F = \frac{YA}{L} x$$

Elastic Potential Energy

$$U = \frac{1}{2} Fx = \frac{1}{2} \left(\frac{YA}{L} x \right) x$$

$$\underline{\underline{U = \frac{1}{2} \frac{YA}{L} x^2}}$$

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MECHANICAL PROPERTIES OF SOLIDS

Q.1 If the potential energy between two molecules is given by $U = -\frac{A}{r^6} + \frac{B}{r^{12}}$, then at equilibrium, separation b/w molecules & potential energy =

Ans $U = -\frac{A}{r^6} + \frac{B}{r^{12}}$

$$F = -\frac{dU}{dr} = -A(-6r^{-7}) + B(-12r^{-13})$$

$$0 = \frac{6A}{r^7} - \frac{12B}{r^{13}}$$

$$\frac{6A}{12B} = \frac{1}{r^6} \Rightarrow r = \left(\frac{2B}{A}\right)^{\frac{1}{6}}$$

$$\therefore U = \frac{-A}{2B/A} + \frac{B}{4B^2/A^2}$$

$$= \frac{-A^2}{2B} + \frac{A^2}{4B} = \underline{\underline{\frac{-A^2}{4B}}}$$

Q.2. A 2kg mass is attached to a spring with spring constant $k = 200 \text{ N/m}$. If the mass is displaced by 0.1 m, potential energy in spring = ?

Ans $U = \frac{1}{2} kx^2$

$$= \frac{1}{2} (200)(0.01) = \underline{\underline{1 \text{ J}}}$$

Q.3. A wire of length L & radius r is clamped at one end. If its other end is pulled by a force F , length increases by l . If the radius of wire & applied force are reduced to half of their original values keeping original length constant, increase in length becomes:

Ans $l = \frac{FL}{AY}$

$$A = \pi r^2$$

$$A' = \frac{\pi r^2}{4} ; F' = \frac{F}{2}$$

$$\therefore l' = \frac{\frac{F}{2}(L)}{\frac{\pi r^2}{4}(Y)} = \frac{FL(4)}{2\pi r^2 Y} = \underline{\underline{\frac{2FL}{\pi r^2 Y}}}$$

$$\therefore \underline{\underline{\text{Answer} \Rightarrow 2 \text{ Times}}}$$

Mechanical Properties Of Solids

Q) A man grows into a giant such that his linear dimensions increase by a factor of 9, density remains same, the stress in leg will change by factor of:

- (A) $1/9$ (B) 81 (C) $1/81$ (D) 9

Sol.

$$\text{initial stress} = \frac{F_0}{A} \left\{ \begin{array}{l} \text{Width \& height of man} \\ \text{as } b \& l \text{ respectively} \end{array} \right\}$$

$$S_0 = \frac{F}{l \times b}$$

dimensions increase by 9 factor

$$S_1 = \frac{F_1}{l' \times b'} = \frac{F_1}{9l \times 9b} = \frac{F_1}{81lb}$$

Since, $m = \rho V$ & $F_0 = mg$

$$F_0 = \rho V g, \text{ so, } S_0 = \frac{\rho V g}{lb}$$

~~At~~ new volume $V_1 = V \times 9^3$ ($V = lbh$ & $V_1 = 9^3 \times lbh$)

$$F_1 = \rho V_1 g = \rho V g \times 9^3$$

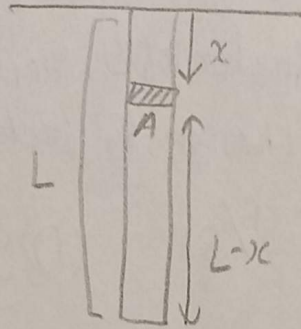
$$\text{new, } S_1 = \frac{\rho V g \times 9^3}{81lb}$$

$$\therefore \frac{S_1}{S_0} = \frac{\frac{\rho V g \times 9^3}{lb \times 81}}{\frac{\rho V g}{lb}} = \frac{9^3}{81} = \boxed{9}$$

$$\boxed{(D) 9}$$

Q) A wire suspended vertically, if elongation due to its own weight is considered, find expression for total elongation. (given, density (ρ), length (L), area (A), Young's modulus)

Sol.



$$F = mg \quad \{ \text{Force} = \text{weight on it} \}$$

$$F = \rho V g$$

$$F = \rho A (L-x) g$$

$$\frac{F}{A} = Y \frac{\Delta L}{L} \quad \{ \Delta L = dx \} \rightarrow d\Delta L = \frac{F dx}{AY}$$

$$d\Delta L = \frac{\rho A (L-x) g dx}{AY} = \frac{\rho g}{Y} (L-x) dx$$

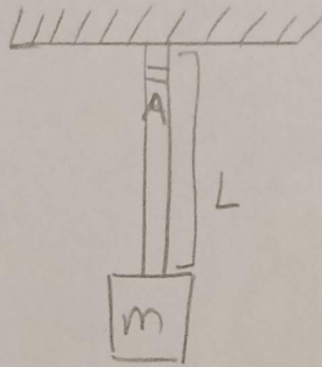
$$\int_0^{\Delta L} d\Delta L = \frac{\rho g}{Y} \int_0^L (L-x) dx$$

$$\Delta L = \frac{\rho g}{Y} \times \left[Lx - \frac{x^2}{2} \right]_0^L$$

$$\Delta L = \frac{\rho g L^2}{2Y}$$

Q) A wire of length L and area A is stretched gradually by hanging a mass m . If mass is suddenly removed, find maximum contraction speed of wire.

Sol.



Elastic Potential Energy:

$$U = \frac{1}{2} F \Delta L \left(\Delta L = \frac{mgL}{AY} \right)$$

$$U = \frac{1}{2} mg \Delta L = \frac{m^2 g^2 L}{2AY}$$

Elastic Potential Energy stored equal to kinetic energy

$$U_{\text{extension}} = K_{\text{normal}}$$

$$\frac{m^2 g^2 L}{2AY} = \frac{1}{2} m v^2$$

$$v = \sqrt{\frac{m g^2 L}{AY}}$$

Properties of Solids

(Q1) Young's Modulus of material of a wire of length 'L' and cross-sectional Area A is Y . If the length of the wire is doubled and cross-sectional area is halved then Young's modulus will be:

Answer:-

$$Y = \frac{FL}{A\Delta L} ; Y' = \frac{F(2L)}{\left(\frac{A}{2}\right)\Delta L} = \frac{2FL \times 2}{A\Delta L} = \frac{4FL}{A\Delta L}$$

When geometry changes, extension ΔL also changes accordingly so the ratio stays the same.

$$\Rightarrow Y' = Y.$$

$\therefore$ Young's Modulus remains unchanged.

(Q2) A wire of length and radius r is clamped at one end. If its other end is pulled by a force F , its length increases by ℓ . If the radius of the wire and the applied force both are reduced to half of their original values keeping original length constant, the increase in length will become:

Answer:-

$$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{\frac{F}{\pi r^2}}{\frac{\Delta L}{L}} = Y \pi r^2 \times \frac{L}{\Delta L} \quad \dots (1)$$

$$\therefore Y = \frac{F/L}{\frac{\pi r^2/4}{\frac{\Delta L}{L}}} = Y \frac{\Delta L}{L} \times 2 \times \frac{\pi r^2}{4}$$

$\therefore$ From (1),

$$Y \pi r^2 \frac{L}{\Delta L} = Y \times \frac{\Delta L}{L} \times \frac{\pi r^2}{2}$$

$$\Rightarrow \boxed{\Delta L = 2L}$$

(Q3) A 100m long wire with a cross-sectional area of $6.25 \times 10^{-4} \text{ m}^2$ and Young's modulus 10^{10} Nm^{-2} is subjected to a 250N load. The elongation (ΔL) in the wire is 4×10^{-3} or (4mm).

$$\therefore Y = \frac{FL}{A\Delta L} \Rightarrow \Delta L = \frac{FL}{AY}$$

$$\therefore \Delta L = \frac{250 \times 100}{(6.25 \times 10^{-4}) \times (10^{10})} = \frac{25000}{6.25 \times 10^6} = 0.004 \text{ m.}$$

Mechanical properties of solid

(Q1) A rod of linear mass density λ and length L is bent to form a ring of radius R . The moment of inertia of the ring about any of its diameter is:-

(a) $\frac{\lambda L^3}{4\pi^2}$ (b) $\frac{\lambda L^3}{4\pi^2}$ (c) $\frac{\lambda L^2}{12}$ (d) $\frac{\lambda L^3}{8\pi^2}$

Answer :- $I = \frac{MR^2}{2} = \frac{\lambda L}{2} R^2 \Rightarrow \frac{\lambda L L^2}{2 \times 4\pi^2} \left[\because R = \frac{L}{2\pi} \right]$

$$\left(= \frac{\lambda L^3}{8\pi^2} \text{ (d)} \right)$$

(Q2) A uniform disc of radius R and mass M is rolling without slipping on a horizontal surface with velocity v . The kinetic energy of disc is:-

(A) $\frac{1}{4} mv^2$ (b) $\frac{1}{2} mv^2$ (c) $\frac{3}{4} mv^2$ (d) Mv^2

Answer :- Total KE = Translational KE + Rotational KE

Moment of inertia $I = \frac{1}{2} Mv^2 + \frac{1}{2} I\omega^2$

$\omega = \frac{v}{R}$

Rotational KE = $\frac{1}{2} \times \frac{1}{2} MR^2 \times \left(\frac{v}{R}\right)^2$

$= \frac{1}{4} mv^2$

Total KE = $\frac{1}{2} mv^2 + \frac{1}{4} mv^2$

$\left(= \frac{3}{4} mv^2 \text{ (c)} \right)$

(Q3) A solid sphere of mass M and radius R is rolling without slipping with a velocity v . Its total kinetic energy is:

(a) $\frac{2}{5} mv^2$ (b) $\frac{7}{10} mv^2$ (c) $\frac{1}{2} mv^2$ (d) Mv^2

Answer:- Moment of Inertia $I = \frac{2}{5} MR^2$
translational KE = $\frac{1}{2} Mv^2$

$$\text{Rotational KE} = \frac{1}{2} I \omega^2 = \frac{1}{2} \cdot \frac{2}{5} MR^2 \cdot \left(\frac{v}{R}\right)^2 = \frac{1}{5} Mv^2$$

$$\text{Total KE} = \frac{1}{2} mv^2 + \frac{1}{5} mv^2 = \frac{5}{10} mv^2 + \frac{2}{10} mv^2 = \frac{7}{10} mv^2 \text{ (b)}$$

option (b) $\frac{7}{10} mv^2$

Mechanical properties of solids

Q.1 The elastic limit of brass is 379 MPa. What should be the minimum diameter of a brass rod if it is to support a 400N load without exceeding its elastic limit?

$$\begin{aligned} \text{Ans: Stress} &= \frac{400 \times 4}{\pi d^2} \\ &= 379 \times 10^6 \text{ N/m}^2 \end{aligned}$$

$$d^2 = (400 \times 4) / (379 \times 10^6 \pi)$$

$$d = 1.15 \text{ mm}$$

Q.2 A man grows into a giant such that his linear dimensions increase by a factor of 9. Assuming that his density remains the same, the stress in the leg will change by a factor of?

$$\text{Stress} = \text{Force} / L^2$$

$$S = (\text{Volume} \times \text{density}) \times \frac{g}{L^2}$$

$$S = L^3 \times \rho g / L^2 = L \rho g$$

Stress $S \propto L$

$$\frac{S_2}{S_1} = \frac{L_2}{L_1} = \frac{9L_1}{L_1} = 9$$

Q.3 Two wires are made of the same material and have the same volume, wire 1 has a cross sectional area A and wire 2 has cross sectional area $3A$. If the length of the wire 1 increases Δx on applying force F , how much force is needed to stretch wire 2 by the same amount?

$$V = V_1 = V_2$$

$$V = A \times L_1 = 3A \times L_2 = L_2 = \frac{L_1}{3}$$

$$Y = \left(\frac{F}{A} \right) \left(\frac{\Delta L}{L} \right)$$

$$F_1 = Y A \left(\frac{\Delta L_1}{L_1} \right), F_2 = Y 3A \left(\frac{\Delta L_2}{L_2} \right)$$

$$\text{Given } \Delta L_1 = \Delta L_2 = \Delta x$$

$$F_2 = Y 3A \left(\frac{\Delta x}{\frac{L_1}{3}} \right) = 9 \frac{Y A \Delta x}{L_1} = 9F_1 = 9F$$

Mechanical properties of solid.

Q. A metallic bar of Young's modulus $0.5 \times 10^{11} \text{ N m}^{-2}$ and coefficient of linear expansion $10^{-5} \text{ } ^\circ\text{C}^{-1}$ length 1m and area of cross-section 10^{-3} m^2 is heated from 0°C to 100°C without expansion or bending.

Sol:

$$\sigma = Y \alpha \Delta T$$

$$F = \sigma A = Y \alpha \Delta T \cdot A$$

$$F = (0.5 \times 10^{11}) (10^{-5}) (100) (10^{-3})$$

$$\Rightarrow 5 \times 10^5 \cdot 100 = 5 \times 10^7$$

$$\Rightarrow 5 \times 10^7 \times 10^{-3} = 5 \times 10^4 \text{ N}$$

$$\boxed{50 \times 10^3 \text{ N}}$$

Q. Copper of fixed volume V is drawn into wire of length l . When this wire is shown to constant force f extension produced is Δl . Which equation is a straight line.

Sol:

$$A = V/l$$

$$\Delta L = \frac{fL}{AY}$$

$$\Delta L = \frac{f \cdot L}{Y(V/l)} = \frac{fL^2}{YV}$$

$$\boxed{\Delta L \propto L^2}$$

Q3 wire of length "L". area of cross section "A" is hanging from a fixed support. The length of wire changes to L_1 when mass "M" is suspended at end.

Sol:

$$Y = \frac{\text{stress}}{\text{strain}} = \frac{F/A}{\Delta L/L} = \frac{FL}{A\Delta L}$$

$$Y = \frac{Mg \cdot L}{A(L_1 - L)}$$

$$Y = \frac{Mg L}{A(L_1 - L)}$$

- Q. A 100m long wire having cross-sectional area $6.25 \times 10^{-4} \text{ m}^2$ & young modulus is 10^{10} Nm^{-2} is subjected to a load of 250N then the elongation in the wire will be: (JEE Main 2023)

Sol: Stress = y strain = $\frac{N}{A} = \frac{y \Delta l}{l}$

$$\Delta l = \frac{Nl}{yA} \Rightarrow \Delta l = \frac{250 \times 100}{10^{10} \times 6.25 \times 10^{-4}}$$

$$\Delta l = 4 \times 10^{-3} \text{ m}$$

- Q. The elastic limit of brass is 379 MPa. What should be the minimum diameter of a brass rod if it is to support a 400N without exceeding its elastic element.

Sol: $P/A = \text{stress}$

$$\frac{400 \times 4}{\pi d^2} = 379 \times 10^6$$

$$d^2 = \frac{1600}{\pi \times 379 \times 10^6} = 1.34 \times 10^{-6}$$

$$d = \sqrt{1.34 \times 10^{-6}} \\ = 1.15 \times 10^{-3} \text{ m}$$

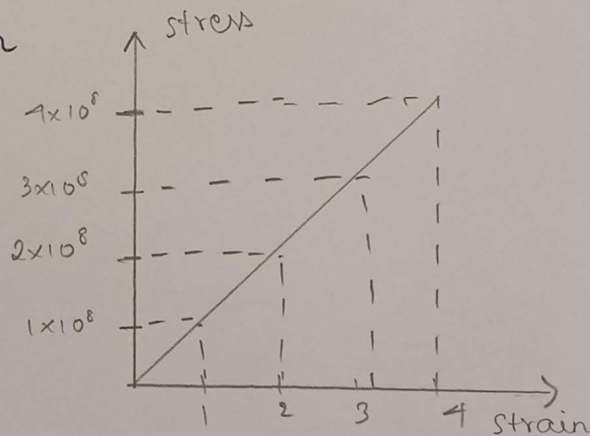
- Q. The stress vs strain graph of a material is shown. Find the Young's modulus of material. (JEE Main 2026)

Sol: Strain = 4
Stress = $4 \times 10^8 \text{ N/m}^2$

$$Y = \text{stress} / \text{strain}$$

$$Y = 4 \times 10^8 / 4$$

$$Y = 10^8 \text{ N/m}^2$$



Q.) A cube has side length 5cm and modulus of rigidity 10^5 N m^{-2} . The displacement produced by a force 10N in the upper face of cube is — mm

Ans

$$L = 0.05 \text{ m}$$

$$\eta = 10^5 \text{ N/m}^2$$

$$F = 10 \text{ N}$$

$$A = 10^{-3} \times 2.5$$

$$\text{Stress} = F/A = 4000 \text{ N m}^{-2}$$

$$\text{Strain} = \frac{\Delta x}{L}$$

$$\eta = \frac{F/A}{\Delta x/L} = \boxed{\Delta x = 2 \text{ mm}}$$

Q.) The young's modulus of steel wire of radius 'r' and length 'l' is γ . If the radius r and length L of the wire are doubled then the value of ' γ '

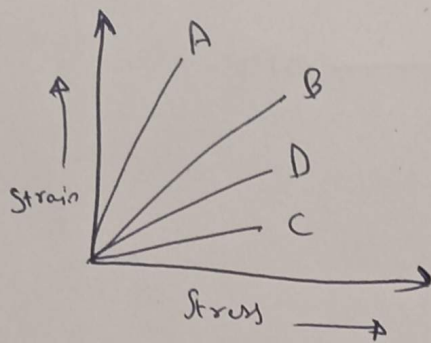
- (A) increases by 2 times (B) reduces by half
(C) remains unchanged. (D) becomes one fourth

Ans $\rightarrow$

Young's modulus is intrinsic property of material of wires

$\gamma \rightarrow$ (C) remains unchanged

Q. The Strain vs stress for materials A, B, C and D is shown. Which material has largest Young's modulus



(A) D

(B) B

(C) C

(D) A

Ans:

(C)

Mechanical Property of Solids.

Q Two wire A and B are made of the same material and have the same length

• Radius A = r

• Radius B = $2r$

Wire A support a mass M

Wire B support a mass $4M$

Find ratio of elongation $\Delta L_A : \Delta L_B$

⇒

$$\Delta L = \frac{FL}{AY}$$

$$\Delta L \propto \frac{F}{A} \quad (\text{since material has same length and material})$$

For wire A:

$$F_A = Mg, \quad A_A = \pi r^2$$

$$\Delta L_A \propto \frac{Mg}{\pi r^2}$$

For wire B:

$$F_B = 4Mg; \quad A_B = \pi (2r)^2 = 4\pi r^2$$

$$\Delta L_B = \frac{4Mg}{4\pi r^2} = \frac{Mg}{\pi r^2}$$

$$\frac{\Delta L_A}{\Delta L_B} = \frac{\frac{Mg}{\pi r^2}}{\frac{Mg}{\pi r^2}} = \frac{1}{1} \Rightarrow \underline{\underline{\Delta L_A : \Delta L_B = 1 : 1}}$$

Q A uniform wire of length L and cross-sectional area A is fixed at the top. Two masses m and $2m$ attached at distance $L/3$ and $2L/3$ from the top respectively.

Find the total elongation of the wire:

$$\Rightarrow F_1 = (m + 2m)g = 3mg$$

$$\text{length} = L/3$$

$$\Delta L_1 = \frac{3mg(L/3)}{AY} = \frac{mgL}{AY}$$

$$F_2 = 2mg$$

$$\text{length} = L/3$$

$$\Delta L_2 = \frac{2mg(L/3)}{AY} = \frac{2mgL}{3AY}$$

$$\Delta L = L_1 + L_2 = \frac{mgL}{AY} + \frac{2mgL}{3AY}$$

$$\boxed{\Delta L = \frac{5mgL}{3AY}}$$

Q. A cube of side 10cm is subjected to uniform pressure P . If its volume decreases by 1cm^3 , find P .

Given bulk modulus $K = 1 \times 10^{11} \text{Pa}$

$\Rightarrow$

$$V = (10)^3 = 1000 \text{ cm}^3$$

$$\frac{\Delta V}{V} = \frac{1}{1000}$$

$$K = \frac{P}{\Delta V/V} = P \times \frac{1}{1000}$$

$$P = 10^{11} \times 10^{-3} = \boxed{10^8 \text{ Pa}}$$

Q1) Determine the volume contraction of a solid copper cube, 10cm on an edge, when subjected to hydraulic pressure $7 \times 10^6 \text{ Pa}$

A) $L = 0.1 \text{ m}$, $p = 7 \times 10^6 \text{ Pa}$, $B = 140 \times 10^9 \text{ Pa}$

$$B = \frac{p}{\frac{\Delta V}{V}}$$

$$\Delta V = \frac{pV}{B}$$

$$= \frac{pL^3}{B}$$

$$= \frac{7 \times 10^6 \times (0.1)^3}{140 \times 10^9} = 5 \times 10^{-8} \text{ m}^3 = 5 \times 10^{-2} \text{ cm}^3 //$$

Q2) A mild steel wire of length 1.0m and cross-sectional area $0.5 \times 10^{-2} \text{ cm}^2$ is stretched, within its elastic limit, horizontally between two pillars. A mass of 100g is suspended from the mid-point of wire. Calculate depression at midpoint

A) Given, $m = 100 \text{ g} = 0.1 \text{ kg}$, $L = 1 \text{ m}$, $A = 0.5 \times 10^{-2} \text{ cm}^2$

$$CD = x$$

$$AB = BD = L = 0.5 \text{ m}$$

$$AD = BD = \sqrt{L^2 + x^2}$$

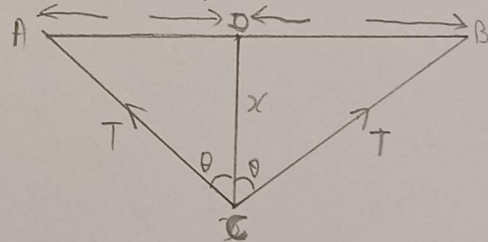
$$\Delta L = AD + BD - AB = 2AD - AB$$

$$= 2\sqrt{L^2 + x^2} - 2L$$

$$= 2L \left(1 + \frac{x^2}{2L^2} \right) - 2L$$

$$= 2L \left(1 + \frac{x^2}{2L^2} - 1 \right) = \frac{x^2}{L}$$

$$\text{Strain} = \frac{x^2}{2L^2}$$



$$2T \cos \theta = mg$$

$$T = \frac{mg}{2 \cos \theta}$$

$$\cos \theta = \frac{x}{\sqrt{l^2 + x^2}} = \frac{x}{l \sqrt{1 + \frac{x^2}{l^2}}} = \frac{x}{l \left(1 + \frac{x^2}{2l^2}\right)}$$

$$(x \ll l):$$

$$1 + \frac{x^2}{2l^2} \sim 1$$

$$\cos \theta = \frac{x}{l}$$

$$T = \frac{mg}{2x}$$

$$\text{stress} = \frac{T}{A} = \frac{mgl}{2Ax}$$

$$y = \frac{mgl}{2Ax} \times \frac{2l^2}{x^2} = \frac{mgl^3}{Ax^3}$$

$$x = \left(\frac{mgl}{AY} \right)^{\frac{1}{3}}$$

$$x = 0.5 \left(\frac{0.1 \times 10}{20 \times 10^9 \times 0.5 \times 10^{-6}} \right)^{\frac{1}{3}} = 0.01074 \text{ m}$$

$$\text{Depression} = 0.01074 \text{ m}$$

Q3) A piece of copper having a rectangular cross-section of $15.2 \text{ mm} \times 19.1 \text{ mm}$ is pulled in tension with $44,500 \text{ N}$ force, producing only elastic deformation. Calculate resulting strain. ($E = 42 \times 10^9 \text{ Nm}^{-2}$)

$$A) l = 19.1 \text{ mm} = 19.1 \times 10^{-3} \text{ m}, b = 15.2 \text{ mm} = 15.2 \times 10^{-3} \text{ m}$$

$$A = lb = 19.1 \times 10^{-3} \times 15.2 \times 10^{-3} = 2.9 \times 10^{-4} \text{ m}^2$$

$$F = 44500 \text{ N}$$

$$\eta = \frac{\text{stress}}{\text{strain}} = \frac{F}{A \text{ strain}}$$

$$\text{strain} = \frac{F}{A \eta} = \frac{44500}{2.9 \times 10^{-4} \times 42 \times 10^9} = 3.65 \times 10^{-3}$$